

# Einstein-Gauss-Bonnet traversable wormholes satisfying the weak energy condition

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In this paper, we explore higher-dimensional asymptotically flat wormhole geometries in the framework of Gauss-Bonnet (GB) gravity and investigate the effects of the GB term, by considering a specific radial-dependent redshift function and by imposing a particular equation of state. This work is motivated by previous assumptions that wormhole solutions were not possible for the  $k = 1$  and  $\alpha < 0$  case, where  $k$  is the sectional curvature of an  $(n - 2)$ -dimensional maximally symmetric space, and  $\alpha$  is the Gauss-Bonnet coupling constant. However, we emphasize that this discussion is purely based on a nontrivial assumption that is only valid at the wormhole throat, and cannot be extended to the entire radial-coordinate range. In this work, we provide a counterexample to this claim, and find for the first time specific solutions that satisfy the weak energy condition throughout the entire spacetime, for  $k = 1$  and  $\alpha < 0$ . In addition to this, we also present other wormhole solutions which alleviate the violation of the WEC in the vicinity of the wormhole throat.

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## I. INTRODUCTION

Wormholes are nontrivial throatlike geometrical structures which connect two parallel universes or distant parts of the same universe. In 1988, Morris and Thorne introduced a family of traversable wormholes [1], where the fundamental ingredient is the flaring-out condition of the wormhole throat. This latter condition, in framework of general relativity (GR), entails the violation of the null energy condition (NEC), which states that  $T_{\mu\nu}k^\mu k^\nu \geq 0$ , where  $T_{\mu\nu}$  is the energy-momentum tensor and  $k^\mu$  is *any* null vector. Matter that violates the NEC is denoted by *exotic matter* [2, 3]. Due to the problematic issue of the violations of the energy conditions [4], several avenues of research have been explored in order to minimize the usage of exotic matter [5]. For instance, it was shown that dynamical spherically symmetric wormholes can satisfy the energy conditions [6] and the averaged energy conditions over timelike or null geodesics for a period of time [7]. Another interesting construction are the thin-shell wormholes, where the exotic matter is restricted to the throat, and therefore minimize its usage [8].

It was also found that higher-dimensional cosmological wormholes [9] and wormholes in modified gravity, involving higher order curvature invariants, can satisfy the energy conditions [10–14], at least at the throat. In fact, in modified gravity, it was shown that matter threading the wormhole throat can be imposed to satisfy all of the energy conditions, and it is the higher order curva-

ture terms, which may be interpreted as a gravitational fluid, that support these nonstandard wormhole geometries. Thus, one is motivated in exploring wormhole geometries in higher-dimensional theories, due to the fact that these alleviate the violation of the energy conditions, at least at the throat. Of particular interest are the  $n$ -dimensional Lorentzian wormhole geometries [15] that were explored in Lovelock gravity [16], which is the most general theory of gravitation in  $n$  dimensions. In contrast to Einstein gravity, it was found that the wormhole throat radius has a lower limit that depends on the Lovelock coefficients, the dimensionality of the spacetime and the shape function. In addition to this, it was shown that the higher order Lovelock terms with negative coupling constants enlarge the region of normal matter near the throat.

Lorentzian wormhole solutions were also investigated in the context of the  $n$ -dimensional Einstein-Gauss-Bonnet (GB) theory of gravitation, which is the second order Lovelock gravity [17]. These wormholes were found to have features depending on the dimensionality of the spacetime,  $n$ , and the GB coupling constant,  $\alpha$ . It was shown that in a large number of cases, the wormhole throat radius is constrained by  $n$  and  $\alpha$ . The possibility of obtaining solutions with normal and exotic matter limited to the vicinity of the throat was also explored. Similar to the situation in GR, the violation of the weak energy condition (WEC) persists for  $\alpha > 0$ . For  $\alpha < 0$ , this condition may or may not be violated depending on the nature of an inequality involving  $|\alpha|$ ,  $n$ , the radius  $r$ , and the wormhole shape function. Dynamic wormhole solutions in the framework of Lovelock gravity with compact extra dimensions were also analysed [18]. It was shown that as the wormhole inflates with the three-dimensional space, the extra dimensions deflate to very

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small, yet nonvanishing scales. In addition to this, it was also shown that the WEC holds for certain ranges of the free parameters of the theory. Further higher dimensional wormhole solutions have been explored, and we refer the reader to [19, 20] for more details.

A thorough analysis of the properties of  $n$ -dimensional static wormhole solutions was investigated in Einstein-Gauss-Bonnet gravity with or without a cosmological constant. The analysis in [17] was generalized in [21] by assuming that the spacetime possessed symmetries corresponding to the isometries of an  $(n-2)$ -dimensional maximally symmetric space with the sectional curvature  $k = \pm 1, 0$ . The metric was assumed to be at least  $C^2$  and the  $(n-2)$ -dimensional maximally symmetric subspace to be compact. The solutions were classified into general relativistic (GR) and non-GR branches, respectively, depending on the existence or absence of the general relativistic limit  $\alpha \rightarrow 0$ . The authors showed that the branch surface in the GR branch coincides with the wormhole throat respecting the dominant energy condition (DEC), otherwise the NEC is violated. On the other hand, in the non-GR branch for  $k\alpha \geq 0$ , it was shown that there is no wormhole solution. In addition to this, it was also shown in the non-GR branch with  $k\alpha \leq 0$  and  $\Lambda \leq 0$ , for the matter field with zero tangential pressure, that the DEC holds at the wormhole throat if the throat radius satisfies a specific inequality. Furthermore, explicit wormhole solutions respecting the energy conditions in the whole spacetime were obtained in the vacuum and dust cases with  $k = -1$  and  $\alpha > 0$ .

We emphasize that despite the fact that wormhole solutions satisfying the energy conditions for the specific case of  $k = -1$  and  $\alpha > 0$ , as mentioned above, no solutions were found for the  $k = 1$  and  $\alpha < 0$ , which was the case extensively explored in Ref. [17]. In the latter, the authors claimed that wormholes solutions were not possible for  $k = 1$  and  $\alpha < 0$ . However, as also mentioned in [21], this discussion is purely based on a nontrivial assumption, which seems to be valid only at the wormhole throat, and cannot be extended throughout the entire range of the radial coordinate. Although, no counterexample was provided in [21], here we provide for the first time a specific solution for wormholes that satisfy the WEC for the specific case of  $k = 1$  and  $\alpha < 0$ .

In addition to this, in all of the above GB wormhole studies, the redshift function is considered to be zero. Here, we relax this assumption and consider a specific radial-dependent choice for the redshift function, which tends to zero at spatial infinity. Indeed, since the GB term has low effects on regions far from throat, one can expect that a non-constant redshift function may contribute to solutions satisfying the energy conditions. Thus, in this paper, we discuss higher dimensional wormhole solutions in the framework of GB gravity and investigate the effects of the GB term, with the presence of  $r$ -dependent redshift functions and considering a specific equation of state, on the satisfaction of the WEC.

The paper is organized as follows: In Sec. II, we give

a brief review of the field equations of GB gravity, and introduce an equation of state in order to solve the field equations. In Sec. III, several wormhole solutions are presented, more specifically, by considering particular choices for the parameters of the theory. In Sec. IV, we summarize and discuss our results.

## II. ACTION AND FIELD EQUATIONS

### A. Action

The action in the framework of GB theory, in the presence of a cosmological constant, is given by

$$I_G = \int d^n x \sqrt{-g} [R - 2\Lambda + \alpha_2 \mathcal{L}_{GB}], \quad (1)$$

where  $n$  is the dimension of the space-time;  $R$  and  $\Lambda$  are the  $n$ -dimensional Ricci scalar and the cosmological constant, respectively;  $\alpha_2$  is the Gauss-Bonnet (GB) coefficient, and the GB term  $\mathcal{L}_{GB}$  is given by

$$\mathcal{L}_{GB} = R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}. \quad (2)$$

In Lovelock theory, for each Euler density of order  $\bar{k}$  in  $n$  dimension space-time, only terms with  $\bar{k} < n$  exist in the equations of motion [22]. Therefore, the solutions of the Einstein-Gauss-Bonnet theory are in  $n \geq 5$  dimensions. Note that the action (1) is derived in the low energy limit of string theory [23].

Now, varying the action (1) with respect to metric, one obtains the field equations

$$G_{\mu\nu} + \alpha_2 \mathcal{G}_{\mu\nu} = T_{\mu\nu}, \quad (3)$$

where  $T_{\mu\nu}$  is the energy-momentum (EM) tensor,  $G_{\mu\nu}$  is Einstein tensor and  $\mathcal{G}_{\mu\nu}$  is the GB tensor, given by

$$\begin{aligned} \mathcal{G}_{\mu\nu} = & 2(-R_{\mu\sigma\kappa\tau}R^{\kappa\tau\sigma}{}_{\nu} - 2R_{\mu\rho\nu\sigma}R^{\rho\sigma} \\ & - 2R_{\mu\sigma}R^{\sigma}{}_{\nu} + RR_{\mu\nu}) - \frac{1}{2}\mathcal{L}_{GB}g_{\mu\nu}. \end{aligned} \quad (4)$$

We use a unit system with  $8\pi G_n = 1$ , where  $G_n$  is the  $n$ -dimensional gravitational constant.

In this work, we consider the  $n$ -dimensional spacetime, by replacing the two-sphere [1] with a  $(n-2)$ -sphere, given by the following line element

$$ds^2 = -e^{2\phi(r)}dt^2 + \frac{dr^2}{1-b(r)/r} + r^2 d\Omega_{n-2}^2, \quad (5)$$

where  $d\Omega_{n-2}^2$  is the metric on the surface of the  $(n-2)$ -sphere.  $\phi(r)$  is denoted the redshift function as it is related to the gravitational redshift; and  $b(r)$  is denoted the shape function because it determines the shape of the wormhole, as can be shown by embedding diagrams [1]. The radial coordinate  $r$  is non-monotonic in that it decreases from  $+\infty$  to a minimum value  $r_0$ , which represents the throat of the wormhole, and then increases

to  $+\infty$ . The shape function at the throat is defined as  $b(r_0) = r_0$ . Note that  $\phi(r)$  should be finite everywhere in order to avoid the presence of an event horizon [1].  $b(r)$  should satisfy the flaring-out condition, i.e.,  $rb' - b < 0$ , so that at the throat we verify the condition  $b'(r_0) < 1$ . The condition  $1 - b(r)/r \geq 0$  is also imposed. Note that although the metric coefficient  $g_{rr}$  becomes divergent at the throat, signalling a coordinate singularity, the proper radial distance  $l(r) = \pm \int_{r_0}^r dr/(1 - b/r)^{1/2}$  is required to be finite everywhere. Thus, the proper distance decreases from  $l = +\infty$ , in the upper universe, to  $l = 0$  at the

throat, and then from zero to  $-\infty$  in the lower universe.

## B. Field equations

The EM tensor is given by  $T_\nu^\mu = \text{diag}[-\rho(r, t), p_r(r, t), p_t(r, t), \dots]$ , where  $\rho(r)$  is the energy density and  $p_r(r)$  and  $p_t(r)$  are the radial and transverse pressures, respectively. Thus, the gravitational field equation (3) provides the following relations

$$\rho(r) = \frac{(n-2)}{2r^2} \left\{ - \left( 1 + \frac{2\alpha b}{r^3} \right) \frac{(b - rb')}{r} + \frac{b}{r} \left[ (n-3) + (n-5) \frac{\alpha b}{r^3} \right] \right\}, \quad (6)$$

$$p_r(r) = \frac{(n-2)}{2r} \left\{ 2 \left( 1 - \frac{b}{r} \right) \left( 1 + \frac{2\alpha b}{r^3} \right) \phi' - \frac{b}{r^2} \left[ (n-3) + (n-5) \frac{\alpha b}{r^3} \right] \right\}, \quad (7)$$

$$p_t(r) = \left( 1 - \frac{b}{r} \right) \left( 1 + \frac{2\alpha b}{r^3} \right) \left[ \phi'' + \phi'^2 + \frac{(b - rb')\phi'}{2r(r-b)} \right] + \left( 1 - \frac{b}{r} \right) \left( \frac{\phi'}{r} + \frac{b - b'r}{2r^2(r-b)} \right) \left[ (n-3) + (n-5) \frac{2\alpha b}{r^3} \right] - \frac{b}{2r^3} \left[ (n-3)(n-4) + (n-5)(n-6) \frac{\alpha b}{r^3} \right] - \frac{2\phi'\alpha}{r^4} \left( 1 - \frac{b}{r} \right) (b - b'r)(n-5), \quad (8)$$

where the prime denotes a derivative with respect to the radial coordinate  $r$ . We define  $\alpha = (n-4)(n-3)\alpha_2$  for notational convenience. We provide below several strategies for solving the field equations.

## C. Strategy of solving the field equations

We now have three equations, namely, the field equations (6)-(8), with the following five unknown functions  $\rho(r)$ ,  $p_r(r)$ ,  $p_t(r)$ ,  $b(r)$  and  $\phi(r)$ . Therefore, in order to determine the wormhole geometry, one can adopt several strategies. For instance, one can apply restrictions on  $b(r)$  and  $\phi(r)$  or on the EM tensor components. It is also common to use a specific equation of state (EOS) relating the EM tensor components, such as, specific equations of state responsible for the present accelerated expansion of the Universe [24] and the traceless EM tensor equations of state [25], amongst others.

In this work, we use an equation of state (EOS) of the form [26]

$$\rho = \omega [p_r + (n-2)p_t]. \quad (9)$$

Using Eq. (9), the trace of the EM tensor can be written as  $T = -\rho + p_r + (n-2)p_t = \rho(1 - \omega)/\omega$ . This EOS will be particularly useful, as for  $\omega = 1$ , it reduces to a traceless EOS,  $T = 0$ , which is usually associated with

the Casimir effect, and that will be extensively explored in the solutions presented below.

Now, substituting  $\rho$ ,  $p_r$  and  $p_t$  in the EOS, one obtains the following differential equation

$$b'(r) = \left\{ 2r^2\omega(n-2)(r-b)(r^3 + 2\alpha b)(\phi'^2 + \phi'') + r\omega\phi'\eta - \xi \right\} / \zeta, \quad (10)$$

with the following definitions

$$\eta = (n-2) \{ 2r^4(n-2) - rb[r^2(2n-5) - 4\alpha] - 2\alpha b^2 \},$$

$$\xi = r^3b(n-2)(n-4)[(n-3)\omega + 1] + \alpha b^2(n-2)(n-7)[(n-5)\omega + 1],$$

and

$$\zeta = r^4(n-2)[(n-3)\omega + 1] + 2\alpha rb(n-2)[\omega(n-5) + 1] - (n-2)\omega r^2\phi' \{ (2\alpha b(-2n+9) + r[4\alpha(n-5) - r^2]) \}.$$

With the EOS given by Eq. (9) in hand, an additional restriction is necessary in order to close the system and solve the field equations. For this purpose, we choose an asymptotically flat redshift function given by

$$e^{2\phi(r)} = \phi_0 + \phi_1 \left( \frac{r_0}{r} \right)^m, \quad (11)$$

where  $\phi_0$  and  $\phi_1$  are dimensionless constants and  $m$  is a positive constant. Note that choosing  $\phi_1 = 0$ , Eq. (11) reduces to the well-known zero tidal force case [1]. As mentioned above, we are interested in analysing solutions that are asymptotically flat, i.e.,  $b(r)/r \rightarrow 0$  and  $\phi(r) \rightarrow 0$  as  $r \rightarrow \infty$ .

### III. WORMHOLE SOLUTIONS

It is well-known that static traversable wormholes in four dimensions violate the energy conditions at or near the wormhole throat in GR [1–3]. These violations are derived from the flaring-out condition of the wormhole throat. On the other hand, the energy conditions can be satisfied in the vicinity of static wormhole throats in

higher dimensional alternative theories [15, 17] and the whole space-time in the case of higher order curvature terms [11], and dynamic wormholes [18]. In the context of the local energy conditions, we examine the weak energy condition (WEC), i.e.,  $T_{\mu\nu}U^\mu U^\nu \geq 0$  where  $U^\mu$  is a timelike vector. For a diagonal EM tensor, the WEC implies  $\rho \geq 0$ ,  $\rho + p_r \geq 0$  and  $\rho + p_t \geq 0$ , simultaneously. Note that the last two inequalities are defined as the NEC.

Using Eqs. (6)-(8), one finds the following relationships

$$\rho + p_r = -\frac{(n-2)}{2r^2} \left[ \frac{(b-rb')}{r} + 2\phi'(b-r) \right] \left( 1 + \frac{2\alpha b}{r^3} \right), \quad (12)$$

$$\begin{aligned} \rho + p_t = & -\frac{(b-rb')}{2r^3} \left( 1 + \frac{6\alpha b}{r^3} \right) + \frac{b}{r^3} \left[ (n-3) + (n-5)\frac{2\alpha b}{r^3} \right] + \phi' \left[ \frac{b-rb'}{2r^2} \left( 1 - \frac{2\alpha b}{r^3}(9-2n) \right) \right. \\ & \left. - \frac{b}{r^2} \left( (n-3) + \frac{2\alpha b}{r^3}(n-5) \right) + \frac{1}{r} \left( (n-3) + \frac{2\alpha b'(n-5)}{r^2} \right) \right] + \left( 1 - \frac{b}{r} \right) \left( 1 + \frac{2\alpha b}{r^3} \right) (\phi'^2 + \phi''), \end{aligned} \quad (13)$$

respectively. One can easily show that for  $\alpha = 0$  and  $\phi' = 0$  the NEC, and consequently the WEC, are violated, due to the flaring-out condition.

Note that at the throat, one verifies

$$(\rho + p_r)|_{r=r_0} = -\frac{n-2}{2r_0^2} (1 - b'_0) \left( 1 + \frac{2\alpha}{r_0^3} \right). \quad (14)$$

Taking into account the condition  $b'_0 < 1$ , and for  $\alpha > 0$ , one verifies the general condition  $(\rho + p_r)|_{r=r_0} < 0$ . For  $\alpha < 0$ , the NEC at the throat is also violated for the range  $r_0 > \sqrt{2|\alpha|}$ . In order to impose  $(\rho + p_r)|_{r=r_0} > 0$ , one needs to consider  $\alpha < 0$ , and the condition  $r_0 < \sqrt{2|\alpha|}$ , which proves that one may have wormholes in GB gravity satisfying the WEC at the throat.

#### A. Solutions violating the WEC

##### 1. Einstein Gravity

In this section, we search for exact solutions in higher-order Einstein gravity ( $\alpha = 0$ ) imposing  $m = (n-3)$  in Eq. (11). For this specific case, Eq. (10) provides the following solution

$$b(r) = \frac{2c_0\phi_0 \left[ \phi_0 + \phi_1 \left( \frac{r_0}{r} \right)^m \right] (1 + \omega m) + \phi_1^2 r_0^m \left( \frac{r_0}{r} \right)^m \omega m}{\phi_0 \left\{ 2\phi_0 r^{(m-1)} [1 + \omega m] + \frac{\phi_1 r_0^m}{r} (2 + \omega m) \right\}}, \quad (15)$$

where  $c_0$  is a constant of integration, and can be determined using the condition  $b(r_0) = r_0$ . It is clear that

the solutions are asymptotically flat, i.e.,  $b(r)/r \rightarrow 0$  as  $r \rightarrow +\infty$ . Note that from Eq. (15) for  $\omega = 0$  and  $\phi_0 = \phi_1$ , one obtains  $b(r) = \frac{c_0}{r^{(n-4)}}$  which presents the Schwarzschild geometry. One also verifies that by choosing  $\phi_1 = 0$ , the wormhole solution of Ref. [27] is obtained.

Choosing  $\omega = 1$  in Eq. (9) leads to a traceless EM tensor solution. In this case, one can obtain wormhole solutions with suitable constants  $\phi_0$  and  $\phi_1$  in order to avoid an event horizon. For instance, in four dimensions Eq. (15) reduces to

$$b(r) = \frac{4c_0\phi_0(\phi_0 \frac{r}{r_0} + \phi_1) + \phi_1 r_0}{\phi_0(4\phi_0 \frac{r}{r_0} + 3\phi_1)}, \quad (16)$$

where  $c_0$  is given by

$$c_0 = \frac{r_0(4\phi_0 - \phi_1)}{4\phi_0}, \quad (17)$$

with  $\phi_1 + \phi_0 > 0$ , to avoid the presence of an event horizon. Using the field equations (6)-(8) one obtains

$$\rho = -\frac{4\phi_1 r_0(c_0\phi_0 + \phi_1 r_0)}{r^2(4\phi_0 r + 3\phi_1 r_0)^2}, \quad (18)$$

$$\rho + p_r = -\frac{16(r\phi_0 + \phi_1 r_0)(c_0\phi_0 + \phi_1 r_0)}{r^2(4r\phi_0 + 3\phi_1 r_0)^2}, \quad (19)$$

$$\rho + p_t = \frac{8\phi_0(c_0\phi_0 + \phi_1 r_0)}{r(4\phi_0 r + 3\phi_1 r_0)^2}. \quad (20)$$

It is clear that the conditions  $\phi_0 + \phi_1 > 0$  and  $\frac{-4\phi_1 r_0(c_0\phi_0 + \phi_1 r_0)}{(4\phi_0 r_0 + 3\phi_1 r_0)^2} < 1$  (which is imposed by  $b'(r_0) < 1$ ) lead to the violation of WEC.

## 2. $\phi_1 = 0$

Since solving the differential equation (10) is too complicated, in general, we will consider restrictions on the redshift function. For instance, consider a constant redshift function, i.e.,  $\phi_1 = 0$  in Eq. (11). Applying these simplified choices, the shape function is given by

$$b(r) = \frac{\left[ -\bar{\omega} \pm \sqrt{\bar{\omega}^2 + 4c_1\alpha(\bar{\omega} - 2\omega)\left(\frac{r}{r_0}\right)^{1-n}} \right] r^3}{2\alpha(\bar{\omega} - 2\omega)}, \quad (21)$$

where we have defined  $\bar{\omega} = \omega(n-3) + 1$  for notational simplicity, and the constant  $c_1$  is given by

$$c_1 = [r_0^2\bar{\omega} + \alpha(\bar{\omega} - 2\omega)] r_0^{-4}. \quad (22)$$

In order to study the behavior of this solution at infinity, we consider the approximation

$$1 - \frac{b(r)}{r} \simeq 1 + \frac{(\bar{\omega} \mp \sqrt{\bar{\omega}^2}) r^2}{2\alpha(\bar{\omega} - 2\omega)} + O\left(\frac{1}{r^{n-3}}\right). \quad (23)$$

There are two classes of wormhole solutions corresponding to the two signs that appear in Eq. (23). It is obvious that these solutions are asymptotically flat if we choose suitable signs in the equation above, namely,  $\bar{\omega} > 0$  for the positive sign, and  $\bar{\omega} < 0$  for the negative sign. We denote these solutions  $b_+$  and  $b_-$ , respectively.

Now, in order to check the WEC, we first investigate the behavior of  $\rho(r)$  for large  $r$ , which is given by the following approximation

$$\rho(r) \simeq -\frac{(n-1)(n-2)\alpha\omega c_1^2}{\bar{\omega}} \left(\frac{r_0}{r}\right)^{2(n-1)} + O\left(\frac{1}{r^{3(n-1)}}\right). \quad (24)$$

Note that  $\rho(r)$  tends to zero as  $r$  increases to infinity. Since  $\rho(r)$  has no real positive root, in order to find its sign, it is sufficient to investigate the sign at infinity. In the case of the  $b_+$  ( $b_-$ ) solution which corresponds to  $\bar{\omega} > 0$  ( $\bar{\omega} < 0$ ), in order to satisfy the WEC, one finds that  $\alpha\omega < 0$  ( $\alpha\omega > 0$ ).

Let us now obtain  $\rho + p_r$  and  $\rho + p_t$  for large  $r$ :

$$\rho + p_r \simeq -\frac{c_1(n-2)}{\bar{\omega}} \left(\frac{r_0}{r}\right)^{n-1} + O\left(\frac{1}{r^{2(n-1)}}\right), \quad (25)$$

and

$$\rho + p_t \simeq \frac{c_1}{\bar{\omega}} \left(\frac{r_0}{r}\right)^{n-1} + O\left(\frac{1}{r^{2(n-1)}}\right). \quad (26)$$

It is clear that both  $\rho + p_r$  and  $\rho + p_t$  tend to zero as  $r$  tends to infinity, with opposite signs. Therefore, in the large  $r$  limit, one of  $\rho + p_r$  or  $\rho + p_t$  is negative and consequently the WEC is violated. However, we show that one can choose suitable values for the constant parameters in order to have normal matter in the vicinity of the throat.

In the following analysis, we consider the traceless EM tensor,  $T = 0$ . This is usually associated to the Casimir

effect, which violates all the energy conditions. Thus, in order to investigate the traceless EM tensor case, we impose  $\omega = 1$ . In this case,  $b_+$  reduces to

$$b_+(r) = \frac{\left[ 2 - n + \sqrt{(n-2)^2 + 4c_1\alpha(n-4)\left(\frac{r}{r_0}\right)^{1-n}} \right] r^3}{2\alpha(n-4)}. \quad (27)$$

The behavior of  $b_+$  in the large  $r$  limit is given by

$$b_+(r) \simeq \frac{c_1 r_0^{(n-1)}}{(n-2)r^{n-4}}, \quad (28)$$

which guarantees the asymptotic flatness of the solution. In order to check the flaring-out condition at the throat, i.e.,  $b'(r_0) < 1$ , one obtains from Eq. (10) that

$$b'(r_0) = -\frac{(n-4)[r_0^2(n-2) + \alpha(n-7)]}{[r_0^2(n-2) + 2\alpha(n-4)]}. \quad (29)$$

Note that in Einstein gravity, where  $\alpha = 0$ , the condition  $b'(r_0) < 1$  is satisfied independent of  $r_0$ . In the case of  $\alpha > 0$ , the flaring-out condition is also satisfied, but for  $\alpha < 0$  it should be carefully checked.

Now, restricting our discussion to the case of the 5-dimensional space-time, one finds that

$$b_+(r) = \frac{(-3r^2 + \sqrt{9r^4 + 4(3r_0^2 + \alpha)\alpha})r}{2\alpha}. \quad (30)$$

The EM tensor components for this solution are given by

$$\rho = \frac{3[(3\kappa - 9r^2)r^2 - 2(3r_0^2 + \alpha)\alpha]}{\alpha r^2 \kappa}, \quad (31)$$

$$\rho + p_r = \frac{9[5(\kappa - 3r^2)r^2 - 4(3r_0^2 + \alpha)\alpha]}{4\alpha r^2 \kappa}, \quad (32)$$

$$\rho + p_t = \frac{(45\kappa - 135r^2)r^2 - 28(3r_0^2 + \alpha)\alpha}{4\alpha r^2 \kappa}, \quad (33)$$

respectively, where  $\kappa = \sqrt{9r^4 + 4(3r_0^2 + \alpha)\alpha}$ . Imposing different values on  $\alpha$ , we plot the quantities  $1 - b(r)/r$ ,  $\rho$ ,  $\rho + p_r$  and  $\rho + p_t$  in Fig. 1. Note that the components of the EM tensor tend to zero as  $r$  tends to infinity. Fig. (1-b) shows that for  $\alpha > 0$  the WEC (and also NEC) is violated in the vicinity of the wormhole throat, but for  $\alpha < 0$ , it can be satisfied near the wormhole throat as it is shown in Fig. (1-a). It can also be seen that  $\rho$  and  $\rho + p_t$  have no real root and therefore are positive everywhere, while  $\rho + p_r$  possesses a real root ( $r_c$ ), where the value  $\rho + p_r$  is positive in the radial region  $r_0 \leq r \leq r_c$ . Thus, one may have normal matter in the radial region  $r_0 \leq r \leq r_c$ .

Figure 2 shows that the increase of  $|\alpha|$  enlarges the normal matter region. Briefly, all of these figures show that it is possible to choose suitable values for the constants in order to have normal matter in the vicinity of the throat.

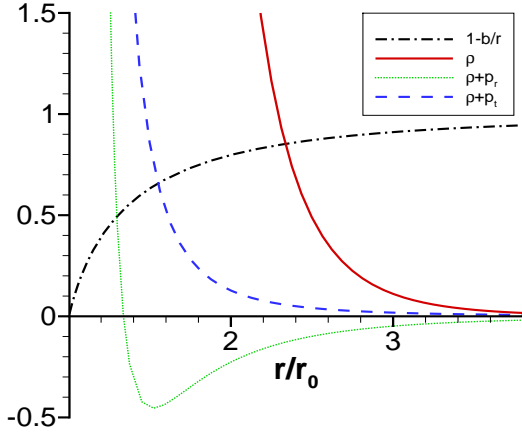
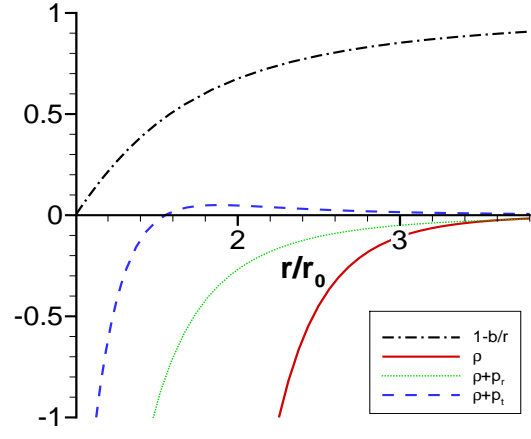
(a)  $\alpha = -1$ (b)  $\alpha = 1$ 

FIG. 1: The specific case of a constant redshift function,  $\phi_1 = 0$ , and for the traceless EOS, with  $T = 0$  ( $\omega = 1$ ), is considered. The behavior of  $1 - b(r)/r$ ,  $10^2 \rho$  (solid),  $\rho + p_r$  (dotted) and  $\rho + p_t$  (dashed) versus  $r/r_0$  for  $n = 5$ , are plotted. It is shown that for  $\alpha < 0$ , the WEC is satisfied near the wormhole throat (a) whereas for  $\alpha > 0$  is not (b).

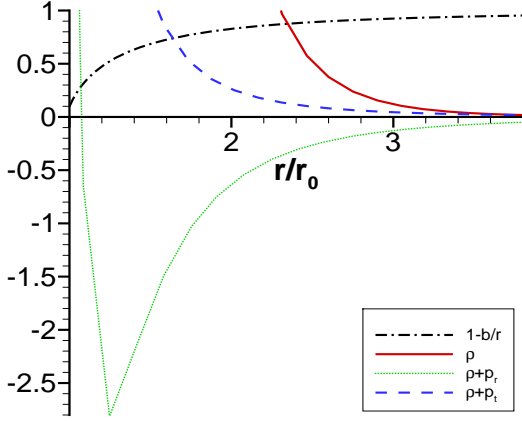
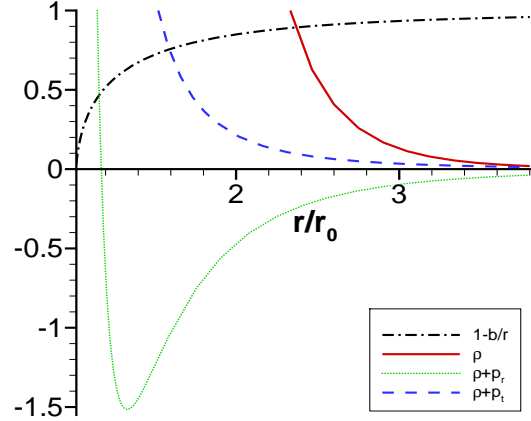
(a)  $\alpha = -1.6$ (b)  $\alpha = -3$ 

FIG. 2: The specific case of a constant redshift function,  $\phi_1 = 0$ , and for the traceless EOS, with  $T = 0$  ( $\omega = 1$ ), is considered. The behavior of  $1 - b(r)/r$  (dot-dash),  $10^3 \rho$  (solid),  $10(\rho + p_r)$  (dotted) and  $10(\rho + p_t)$  (dashed) versus  $r/r_0$  for  $n = 5$ , are plotted. It is shown that the region of normal matter in the vicinity of the thoroat enlarges as the value of  $|\alpha|$  increases.

## B. Solutions satisfying the WEC

In this section, the equations are solved for the full redshift function of the form presented in Eq. (11). Since finding exact analytical solutions is extremely difficult, we consider a simplifying assumption of  $\omega = 1$  so that the EOS reduces to a traceless energy momentum tensor,  $T = 0$ . One may now choose specific constant parameters

so that the solutions are asymptotically flat. Although one cannot find explicit solutions for the shape function, the numerical solutions are plotted in Figs. 3 and 4. We verify that for these choices, the quantity  $b(r)/r$  tends to zero at spatial infinity, and the behavior of  $1 - b(r)/r$  is plotted in the figures. Note that all of the quantities  $\rho(r)$ ,  $\rho(r) + p_r(r)$  and  $\rho(r) + p_t(r)$  are positive throughout the spacetime, implying that the WEC is satisfied for all

values of  $r$ .

Note that Bawal and Kar [17] have claimed that it is not possible to find wormhole solutions of the GB field equations with normal matter everywhere. However, this discussion is based on the positivity of the factor  $[(b - rb')/r + 2\phi'(b - r)]$  [see Eq. (12)] throughout spacetime. This factor is indeed positive at the throat

as shown above, but this behaviour is not guaranteed to be positive for the entire range of the radial coordinate. The solutions presented in Figs. 3 and 4 provide a counterexample to the claim in [17].

Another condition that needs to be satisfied is the condition  $b'(r_0) < 1$  at the throat. We verify that the quantity  $b'_0$  is given by [see Eq. (10)]:

$$b'(r_0) = -\frac{2\phi_0(n-4)[\alpha(n-7) + r_0^2(n-2)] + \phi_1[(n-2)(n-4) + m][2\alpha + r_0^2]}{2\phi_0[2\alpha(n-4) + r_0^2(n-2)] + \phi_1[2\alpha(2(n-2) - m - 8) + r_0^2(2(n-2) - m)]}. \quad (34)$$

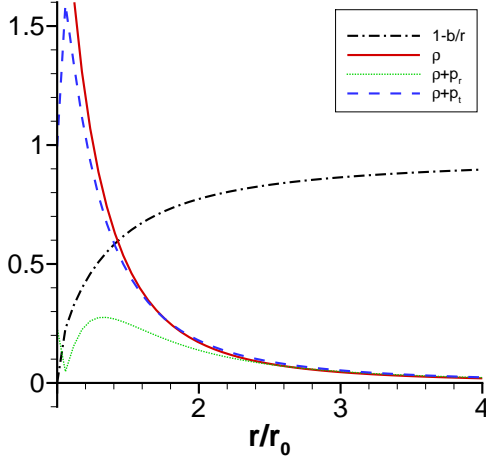


FIG. 3: The behavior of  $1-b(r)/r$  (dot-dash),  $\rho$  (solid),  $\rho + p_r$  (dotted) and  $\rho + p_t$  (dashed) versus  $r/r_0$  for  $\alpha = -1.54$  with  $\omega = 1$  ( $T = 0$ ),  $m = 0.3$ ,  $\phi_0 = 1$ ,  $\phi_1 = -0.96$ ,  $n = 7$  ( $b'(r_0) = -31.59 < 1$ ). This solution satisfies the WEC.

One can find the values of  $b'(r_0)$  for these numerical solutions in the caption of Figs. 3 and 4.

#### IV. SUMMARY AND CONCLUSIONS

In this paper, we have explored higher-dimensional asymptotically flat wormhole solutions in the framework of Gauss-Bonnet (GB) gravity by considering a specific choice for a radial-dependent redshift function and by imposing a particular equation of state. We have shown explicitly that the WEC is satisfied at the throat by considering a negative Gauss-Bonnet coupling constant, i.e.,  $\alpha < 0$ , and in which the wormhole throat is constrained by the following condition  $r_0^2 < 2|\alpha|$ . This confirms previous results outlined in [17]. Furthermore, we have briefly presented solutions in higher-dimensional Einstein gravity,  $\alpha = 0$ , for a specific radial-dependent redshift func-

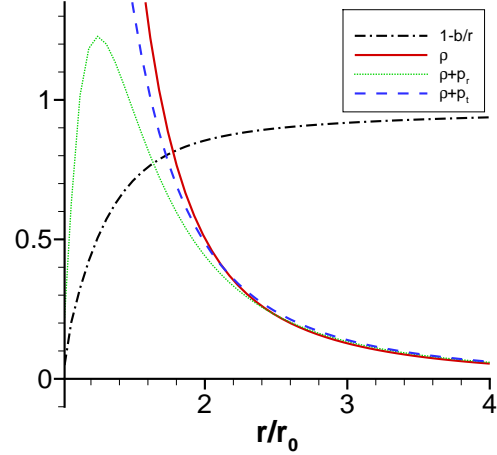


FIG. 4: The behavior of  $1-b(r)/r$  (dot-dash),  $\rho$  (solid),  $\rho + p_r$  (dotted) and  $\rho + p_t$  (dashed) versus  $r/r_0$  for  $\alpha = -1.18$  with  $\omega = 1$  ( $T = 0$ ),  $m = 0.28$ ,  $\phi_0 = 1$ ,  $\phi_1 = -0.99$ ,  $n = 11$  ( $b'(r_0) = -49.35 < 1$ ). This solution satisfies the WEC.

tion. Furthermore, we have considered a constant redshift function and shown specifically that, for  $\alpha < 0$ , one may have normal matter in a determined radial region  $r_0 \leq r \leq r_c$ , and that the increase of  $|\alpha|$  enlarges the normal matter region.

However, the main motivation of this work resides in finding solutions that satisfy the WEC throughout the entire spacetime. We have been intrigued by previous assumptions claiming that wormholes solutions were not possible for the  $\alpha < 0$  case [17]. However, we agree with the discussion in [21], in that the nontrivial assumption discussed in [17], which is valid only at the wormhole throat, cannot be extended throughout the entire range of the radial coordinate. In this work, we provided a counterexample to this claim, and found for the first time solutions that satisfy the WEC throughout the entire spacetime. In this context, it would be interesting to extend the analysis carried out in third order Lovelock

gravity considered in [15], where it was shown that a negative third order coupling constant enlarges the radius of the region of normal matter relative to the second order theory, and perhaps it may be possible to find solutions that satisfy the WEC throughout the entire spacetime. Work along these lines is presently underway.

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- [1] M.S. Morris and K.S. Thorne, *Am. J. Phys.* **56**, 395 (1988).
  - [2] S. Kar, N. Dadhich and M. Visser, *Pramana* **63**, 859 (2004).
  - [3] D. Hochberg and M. Visser, *Phys. Rev. D* **56**, 4745 (1997).
  - [4] C. Barcelo and M. Visser, *Int. J. Mod. Phys. D* **11**, 1553 (2002); F. S. N. Lobo and M. Visser, *Class. Quant. Grav.* **21**, 5871 (2004).
  - [5] M. Visser, S. Kar and N. Dadhich, *Phys. Rev. Lett.* **90**, 201102 (2003).
  - [6] S. Kar and D. Sahdev, *Phys. Rev. D* **53**, 722 (1996); A. V. B. Arellano and F. S. N. Lobo, *Class. Quant. Grav.* **23**, 5811 (2006); M. Cataldo, P. Meza, and P. Minning, *Phys. Rev. D* **83**, 044050 (2011).
  - [7] F. J. Tipler, *Phys. Rev. D* **17**, 2521 (1978).
  - [8] E. Poisson and M. Visser, *Phys. Rev. D* **52**, 7318 (1995); E. F. Eiroa and G. E. Romero, *Gen. Rel. Grav.* **36**, 651 (2004); F. S. N. Lobo and P. Crawford, *Class. Quant. Grav.* **21**, 391 (2004); F. S. N. Lobo and P. Crawford, *Class. Quant. Grav.* **22**, 4869 (2005); F. S. N. Lobo, *Gen. Rel. Grav.* **37**, 2023 (2005); E. F. Eiroa and C. Simeone, *Phys. Rev. D* **71**, 127501 (2005); M. Thibeault, C. Simeone and E. F. Eiroa, *Gen. Rel. Grav.* **38**, 1593 (2006); M. G. Richarte and C. Simeone, *Phys. Rev. D* **76**, 087502 (2007) [Erratum-ibid. *D* **77**, 089903 (2008)]; E. Gravanis and S. Willison, *Phys. Rev. D* **75**, 084025 (2007); M. G. Richarte and C. Simeone, *Int. J. Mod. Phys. D* **17**, 1179 (2008); S. H. Mazharimousavi, M. Halilsoy and Z. Amirabi, *Phys. Rev. D* **81**, 104002 (2010); C. Simeone, *Phys. Rev. D* **83**, 087503 (2011); N. M. Garcia, F. S. N. Lobo and M. Visser, *Phys. Rev. D* **86**, 044026 (2012); M. G. Richarte, *Phys. Rev. D* **87**, 067503 (2013); Z. Amirabi, M. Halilsoy and S. Habib Mazharimousavi, *Phys. Rev. D* **88**, no. 12, 124023 (2013); M. Bouhmadi-López, F. S. N. Lobo and P. Martín-Moruno, *JCAP* **1411**, no. 11, 007 (2014).
  - [9] M. Kord Zangeneh, F. S. N. Lobo and N. Riazi, *Phys. Rev. D* **90**, 024072 (2014).
  - [10] F. S. N. Lobo, *Phys. Rev. D* **75** (2007) 064027; F. S. N. Lobo, *Class. Quant. Grav.* **25** (2008) 175006; F. S. N. Lobo and M. A. Oliveira, *Phys. Rev. D* **80** (2009) 104012; N. M. García and F. S. N. Lobo, *Phys. Rev. D* **82** (2010) 104018; N. Montelongo García and F. S. N. Lobo, *Class. Quant. Grav.* **28** (2011) 085018; C. G. Boehmer, T. Harko and F. S. N. Lobo, *Phys. Rev. D* **85** (2012) 044033; S. Capozziello, T. Harko, T. S. Koivisto, F. S. N. Lobo and G. J. Olmo, *Phys. Rev. D* **86** (2012) 127504.
  - [11] T. Harko, F. S. N. Lobo, M. K. Mak and S. V. Sushkov, *Phys. Rev. D* **87** (2013) 067504.
  - [12] A. G. Agnese and M. La Camera, *Phys. Rev. D* **51**, 2011 (1995); K. K. Nandi, A. Islam and J. Evans, *Phys. Rev. D* **55**, 2497 (1997); X. Yue and S. Gao, *Phys. Lett. A* **375**, 2193 (2011); F. S. N. Lobo and M. A. Oliveira, *Phys. Rev. D* **81**, 067501 (2010); S. V. Sushkov and S. M. Kozyrev, *Phys. Rev. D* **84**, 124026 (2011).
  - [13] L. A. Anchordoqui and S. E. P. Bergliaffa, *Phys. Rev. D* **62**, 067502 (2000); K. A. Bronnikov and S. W. Kim, *Phys. Rev. D* **67**, 064027 (2003); M. La Camera, *Phys. Lett. B* **573**, 27 (2003).
  - [14] V. Dzhunushaliev and D. Singleton, *Phys. Rev. D* **59**, 064018 (1999); J. Ponce de Leon, *JCAP*, 0911:013 (2009).
  - [15] M. H. Dehghani and Z. Dayyani, *Phys. Rev. D* **79**, 064010 (2009).
  - [16] D. Lovelock, *J. Math. Phys. (N.Y.)* **12**, 498 (1971).
  - [17] B. Bhawal and S. Kar, *Phys. Rev. D* **46**, 2464 (1992).
  - [18] M. R. Mehdizadeh and N. Riazi, *Phys. Rev. D* **85**, 124022 (2012).
  - [19] G. Dotti, J. Oliva, and R. Troncoso, *Phys. Rev. D* **75**, 024002 (2007); G. Dotti, J. Oliva and R. Troncoso, *Phys. Rev. D* **76**, 064038 (2007); M. H. Dehghani and S. H. Hendi, *Gen. Rel. Grav.* **41**, 1853 (2009).
  - [20] A. Chodos and S. Detweiler, *Gen. Rel. Grav.* **14**, 879 (1982); P. Kanti, B. Kleihaus and J. Kunz, *Phys. Rev. Lett.* **107**, 271101 (2011); Farook Rahaman, P. K. F. Kuhfittig, Saibal Ray, Safiqul Islam, *Phys. Rev. D* **86**, 106010 (2012); Takashi Torii and Hisa-aki Shinkai, *Phys. Rev. D* **88**, 064027 (2013).
  - [21] H. Maeda and M. Nozawa, *Phys. Rev. D* **78**, 024005 (2008).
  - [22] M. H. Dehghani, N. Bostani, and A. Sheykhi, *Phys. Rev. D* **73**, 104013 (2006).
  - [23] D. J. Gross and E. Witten, *Nucl. Phys. B* **277**, 1 (1986); R. R. Metsaev and A. A. Tseytlin, *Phys. Lett. B* **191**, 354 (1987); C. G. Callan, R. C. Myers, and M. J. Perry, *Nucl. Phys. B* **311**, 673 (1988); R. C. Myers, *Phys. Rev. D* **36**, 392 (1987).
  - [24] S. Sushkov, *Phys. Rev. D* **71**, 043520 (2005); F. S. N. Lobo, *Phys. Rev. D* **71**, 084011 (2005); F. S. N. Lobo, *Phys. Rev. D* **73**, 064028 (2006); F. S. N. Lobo, *Phys. Rev. D* **75**, 024023 (2007); A. DeBenedictis, R. Garattini and F. S. N. Lobo, *Phys. Rev. D* **78**, 104003 (2008); F. S. N. Lobo, F. Parsaei and N. Riazi, *Phys. Rev. D* **87**,



- no. 8, 084030 (2013).
- [25] S. Kar and D. Sahdev, Phys. Rev. D **52**, 2030 (1995); E. Ebrahimi and N. Riazi, Phys. Rev. D **81**, 024036 (2010).
- [26] L. A. Anchordoqui, S. E. Perez Bergliaffa and D. F. Torres, Phys. Rev. D **52**, 5226 (1997).
- [27] M. Cataldo, P. Salgado and P. Minning, Phys. Rev. D **66**, 124008 (2002).
- [28] M. S. R. Delgaty and R. B. Mann, Int. J. Mod. Phys. D **4**, 231 (1995); F. S. N. Lobo, Class. Quant. Grav. **21**, 4811 (2004).